7. Effective Field Theory

You find yourself transported to a parallel universe in which the supertheory has broken in a different pattern. After attending parallel Pitt for a while you learn that the parallel Standard Model (PSM) is a ϕ^4 theory and that this theory suffers from the 'unstable vacuum problem', namely the ϕ^4 coupling is negative. You realise that this theory is likely a low energy effective theory that emerges from a supertheory:

$$\mathcal{L}_{\rm super} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{1}{2} M^2 \Phi^2 - \frac{\kappa}{2}\phi^2 \Phi,$$

where m and κ are much smaller than M.

(1) Astound the locals by computing the coupling constant of the PSM in terms of the super model. Is the coupling perturbative?

(2) Continue to astound the locals by computing the one loop corrections to the normalisation and mass of the phion (a_1 and b_1). Parameterise the PSM as

$$\mathcal{L}_{PSM} = \frac{1}{2}(1+a_1)(\partial_\mu \phi)^2 - \frac{1}{2}(1+b_1) m^2 \phi^2 - \frac{1}{4!}(c_0 + c_1)\phi^4.$$

Renormalise both theories using $\overline{\rm MS}$. Match the theories at the scale $\mu = M$. Verify that infrared logs cancel in your expressions for a_1 and b_1. Is the correction to the phion mass 'natural' or 'unnatural'? *Hint:* You will need to compute a logarithmic Feynman parameter integral. The p^2/M^2 term can be treated as perturbative immediately. The m^2/M^2 term is a bit more problematic since a naive perturbative treatment leads to a spurious infrared divergence (like in the Higgs contribution to the muon anomalous moment). But you can do this integral exactly.

(3) Write all diagrams that are required to compute the one-loop correction to c_0 (called c_1 above).