

## QFT — Some QED: assignment 8

### 1. photon propagator

(i) Show that

$$(\partial_x^2 g_{\mu\rho} - (1 - \lambda)\partial_\mu^x \partial_\rho^x) \langle 0 | T A^\rho(x) A^\nu(y) | 0 \rangle = i g_\mu^\nu \delta^{(4)}(x - y)$$

using the equations of motion, the definition of time ordering, and  $[A_\mu(x, t), \Pi_\nu(y, t)] = i g_{\mu\nu} \delta^{(3)}(x - y)$ .

(ii) Show that

$$\langle 0 | T A^\rho(x) A^\nu(y) | 0 \rangle = -i \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot (x-y)} \left[ \frac{g^{\rho\nu}}{k^2 + i\epsilon} + \frac{(1 - \lambda)}{\lambda} \frac{k^\rho k^\nu}{(k^2 + i\epsilon)^2} \right]$$

by inverting the equation of part (i) and using covariance arguments.

### 2. Møller Scattering

Compute the differential cross section for  $e^- e^- \rightarrow e^- e^-$  in the centre of mass frame to leading order. Express your answer in terms of the centre of mass energy  $E$ , the electron mass  $m$ , and  $\sin \theta$  where  $\theta$  is the centre of mass scattering angle.

### 3. $J/\psi$ decay constant

The  $J/\psi$  is a charm-anticharm vector meson whose discovery confirmed the reality of quarks to many physicists. Define the  $J/\psi$  decay constant,  $f_\psi$ , via the relation

$$\langle \psi(p, \lambda) | \bar{c} \gamma_\mu c | 0 \rangle = f_\psi m_\psi \epsilon_\mu^*(p, \lambda)$$

where  $p$  and  $\lambda$  are the  $J/\psi$ 's four-momentum and polarisation respectively. Use this definition to compute the decay rate for  $J/\psi \rightarrow e^+ e^-$ . Write your answer in terms of  $\alpha$ , the charge of the charm quark ( $= Qe$ ), the decay constant, and the  $J/\psi$  and electron masses.