QFT — More Dirac Basics: assignment 6

1. **Parity**

(i) Give the explicit transformation properties of all bilinears under parity.

(ii) People often wonder how the reaction $\pi \to \mu\nu$ can proceed since the pion has $J = 0$ while the W is a vector ($J = 1$) particle. Explain. Note that the current associated with the W is proportional to $V^\mu - A^\mu = \bar{\psi}\gamma^\mu(1 - \gamma_5)\psi$.

2. **Bilinears**

(i) Prove that the operator $\bar{\psi}\psi$ is a scalar by establishing the quantum numbers of the state $\int d^3x\bar{\psi}(\vec{x})\psi(\vec{x})\,|0\rangle$ in the nonrelativistic limit. Work in the rest frame of the created particle.

(ii) Do the same for the vector current. What happens for $\int d^3x\bar{\psi}\gamma^0\psi\,|0\rangle$?

3. **Chirality and Helicity**

(i) Show that chirality is not a good quantum number for a massive fermion.

(ii) Show that helicity is conserved for a massive fermion.

(iii) Show that $\gamma_5 u = \Sigma \cdot \hat{p}u$ in the high energy limit where $\Sigma = diag(\sigma, \sigma)$. Thus chirality and helicity are the same in the high energy limit.

4. **Foldy-Wouthuysen Transformation**

(i) Confirm the relations $[\beta\mathcal{O}, \mathcal{E}] = \beta[\mathcal{O}, \mathcal{E}]$, $[\beta\mathcal{O}, \mathcal{O}] = 2\beta\mathcal{O}^2$, $[\mathcal{O}, \beta] = -2\beta\mathcal{O}$, $[\beta\mathcal{O}, \beta\mathcal{E}] = -\{\mathcal{O}, \mathcal{E}\}$, $[\beta\mathcal{O}, \beta\mathcal{O}'] = -[\mathcal{O}, \mathcal{O}']$.

(ii) Obtain the F-W Dirac equation. You may follow the description in Itzykson and Zuber.