QFT — assignment 3: Klein-Gordon Preliminaries

NB: "Klein-Gordon" after Oskar Klein and Walter Gordon.
"Clebsch-Gordan" after Alfred Clebsch and Paul Gordan.

1. **Complex Klein-Gordon Fields**

Obtain expressions for the Hamiltonian, field momentum, and conserved charge in terms of fields and particles for free charged bosons.

2. **causality**

Establish that $G_R(x) = -\int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik\cdot x}}{(k^0+i\epsilon)^2 - \vec{k}^2 - m^2}$ vanishes for $t < 0$. What boundary condition (pole prescription) causes G to vanish for $t > 0$?

3. **some relativistic preliminaries**

(i) prove that $\int \frac{d^3k}{(2\pi)^3} \frac{1}{2E(k)}$ is Lorentz invariant.
(ii) prove that $\int \frac{d^3k}{2E(k)} = \int d^4k \delta(k^2 - m^2)\theta(k_0)$. Take $E(k) = \sqrt{m^2 + k^2}$.

4. **Green's Functions**

Prove that $i\Delta_F(x - y) = \langle 0|T[\phi(x)\phi^\dagger(y)]|0\rangle$ is a Green's function for the Klein-Gordon equation by using its definition (given here), $\partial_t\theta(t) = \delta(t)$, and the commutation relation $[\dot{\phi}^\dagger(x), \phi(y)] = -i\delta(x - y)$.